Graphs of Origami

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In this paper we analyze the structure of origami mathematically and present the structure in graphs. When an origami is folded, the faces constituting the origami are divided by the fold line and some faces are rotated and stacked on the other faces. Each face is modeled as a bounded plane with no thickness and with two sides. The origami is then modeled as a structure $(\Pi, \backsim, \succ)$, where Π is a finite set of faces that constitute an origami and $\backsim$ and $\succ$ are binary relations on Π. The relation $\backsim$ is an adjacency relation and relates a given face to its neighborhood. $\succ$ is the relation between overlapping faces that "face" each other. When we distinguish the two binary relations by types and let $R = \backsim \cup \succ$, we have a structure (Π, R). The structure (Π, R) is amenable to a graph, in particular to a typed graph. Furthermore, with a slight concretization of the face structure to the convex polygon, we can obtain a typed hyper graph. Origami fold can be viewed as a graph transformation. Successive graph transformations result in a new graph, which can be seen as the graph of the (final) origami. Figure 1 shows an example. The top picture shows an origami crane created by our Eos system. For clarity, we separate the graph into two untyped graphs $(\Pi, \backsim)$ and $(\Pi, \succ)$, and show them separately. The middle one is the graph $(\Pi, \backsim)$ and the bottom one is the graph $(\Pi, \succ)$. The former may be expected and familiar since it is isomorphic to the crease pattern of the complete development of the crane origami. The latter is somewhat surprising and it can be said that the origami crane has the spine as illustrated here.

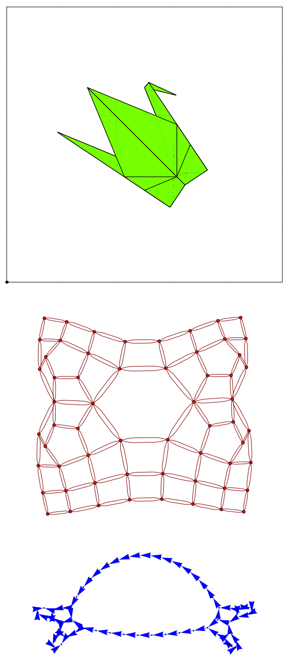

Figure 1: Graphs of Origami Crane