Universal Hinge Patterns to Fold Orthogonal Shapes

Nadia M. Benbernou* Erik D. Demaine*†
Martin L. Demaine* Aviv Ovadya*

An early result in computational origami is that every polyhedral surface can be folded from a large enough square of paper [2]. But each such folding uses a different crease pattern. Can one design a hinge pattern that can be folded into various different shapes?

Our motivation is developing programmable matter out of a foldable sheet. The idea is to statically manufacture a sheet with specific hinges that can be creased in either direction, and then dynamically program how much to fold each crease in the sheet. Thus a single manufactured sheet can be programmed to fold into anything that the single hinge pattern can fold.

We prove a universality result: a single $n \times n$ hinge pattern can fold into all face-to-face gluings of $O(n)$ unit cubes. Thus, by setting the resolution n sufficiently large, we can fold any 3D solid up to a desired accuracy.

The proof is by construction: we describe an algorithm for generating the crease pattern for a given polycube. We also show how an implementation of the algorithm can be used to automate experimentation and design of geometric paper origami using a cutting plotter or laser cutter to score the paper.

We describe three hinge-pattern variants, which are equivalent in order of growth properties, but have different applications. One robotics-motivated variant uses a *box-pleated* lattice—only $90°$ and $45°$ angles. Another variant is more size efficient and symmetrical but requires additional hinges with $\arctan(\frac{1}{2})$ angles and is thus less efficient in number of hinges. A final robotics-motivated variant is as efficient as the $\arctan(\frac{1}{2})$ angle variant, and uses only $90°$ and $45°$ angles, but also requires a regular pattern of slits in the paper. Figure 1 shows our primary gadget for each method, and Figure 2 shows paper examples. For details on the first method, see [1].

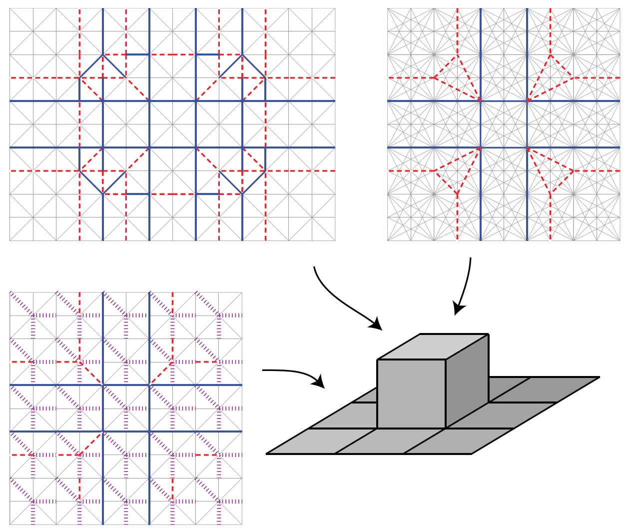

Figure 1: Gadgets for proving universality of the three types of tilings: box–pleat, $\arctan(\frac{1}{2})$, and slit.

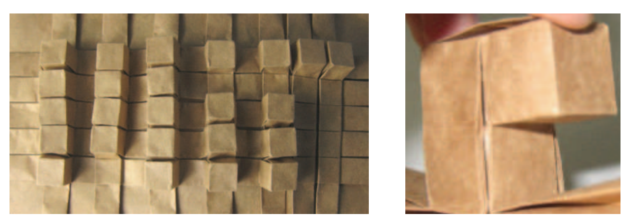

Figure 2: Examples of the algorithm applied to paper.

*MIT Computer Science and Artificial Intelligence Laboratory, 32 Vassar St., Cambridge, MA 02139, USA, {nbenbern, edemaine,mdemaine,avivo}@mit.edu

†Partially supported by NSF CAREER award CCF-0347776, DOE grant DE-FG02-04ER25647, and AFOSR grant FA9550-07-1-0538.

References

[1] N. Benbernou, E. D. Demaine, M. L. Demaine, and A. Ovadya. A universal crease pattern for folding orthogonal shapes. arXiv:0909.5388, September 2009. http://arxiv.org/abs/0909.5388.

[2] E. D. Demaine, M. L. Demaine, and J. S. B. Mitchell. Folding flat silhouettes and wrapping polyhedral packages: New results in computational origami. *Computational Geometry: Theory and Applications*, 16(1):3–21, 2000.