4D Origami Folds: Generalizing the Axioms to Higher Dimensions

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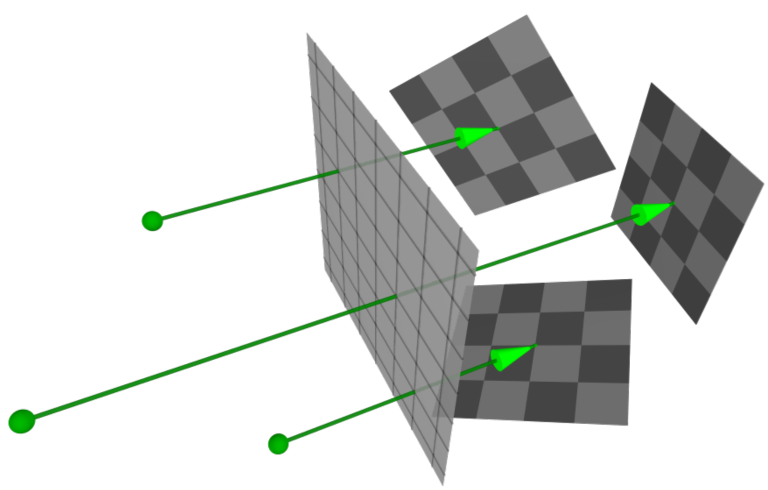
The seven Huzita-Justin axioms of origami construction are a well-accepted set of rules that describe what folds are mathematically possible on a 2-dimensional sheet of paper. Robert Lang has described these folds thoroughly and proved that they comprise all possible 3D folds [1]. At 4OSME, Lang also generalized the axioms by considering two simultaneous folds, and showed that this is more powerful [2]. He also generalized such multifolds to the algebraic closure of the rationals (as did Chow and Fan).

However, we can propose another avenue of generalization: higher dimensions. In "4-dimensional origami," we begin with a unit cube (instead of a unit square) as "paper," and create "fold planes" (instead of fold lines). We have points, lines, and planes (instead of only points and lines) as features on the paper, and these features can be brought into alignment by considering their reflections across fold planes ("4D folds").

It is natural to ask how many kinds of these 4-dimensional axioms/folds exist. Although the exact count depends on arbitrary definitions, it is possible to generalize Lang's proof in [1] and apply the idea of degrees of freedom to enumerate the different possible 4D folds (on the order of over a dozen): we list all the possible alignments and their degrees of freedom, and calculate which combinations produce valid folds. While many of the resulting axioms are essentially equivalent to 3D folds, the extra dimension does allow for some interesting manipulations, such as aligning three points each onto a corresponding plane (see image), which can have up to 7 solutions.



[1] http://www.langorigami.com/science/hha/origami_constructions.pdf
[2] Roger C. Alperin and Robert J. Lang. "One-, Two-, and Multi-Fold Origami Axioms." *Origami*^{4}, p. 371-391. Natick, MA: AK Peters, 2009.